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Discussion Papers in
Economics and Econometrics

Title: Dynamic Squeezing: Marriage and Fertility in France After World War One

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No. 1305

ISSN 0966-4246

Dynamic Squeezing: Marriage and Fertility in France After World War One[*]

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July 1, 2013

Abstract

Unmarried people undoubtedly differ in their preferences for marriage, and such differences are likely to be linked to their preferences for children. We propose a model of people searching for marriage partners in which ageing and fertility propensities determine marriage probabilities. We apply our model to a quantitative analysis of the post-war marriage boom that began in France in 1918. We find that wartime shocks to the marriage market are perpetuated across generations and cause persistent increases in marital birthrates. We also find that permanent heterogeneity in women's propensity for children accounts for most of the increase in marriage relative to trend. We sketch out other applications, notably the impact of the one-child policy in China on marriage rates.

Keywords: Family Economics, Household Formation, Marriage, Fertility.

JEL Classification: D10, E13, J12,J13,and O11.

[*]PRELIMINARY AND INCOMPLETE. For helpful comments, we thank Marco Gomes-Mella and seminar participants at Southampton.

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1 Introduction

The First World War, which ended in November 1918, significantly disrupted the demographic processes of European countries. In France, which saw a large share of the fighting, and suffered a long occupation, marriage rates and marital fertility fell by about 50% during the war. The end of the war saw both marriage and marital birth rates peak dramatically above their pre-war trends. What is more surprising however is the duration of the disruption; marriage rates did not fall back to their pre-war trend until the late 1920s, and marital fertility remained markedly above its pre-war trend well into the 1930s.[1]

We stress the protracted nature of the post-war adjustment because we take this as evidence of the importance of frictions in the matching process. In our empirical analysis below, we show evidence that the disruption of the marriage and birth patterns extends to the younger, non-mobilized cohorts more than ten years after the end of the war [2]. While frictions in pairwise transactions are often asserted to be important in economics, Burdett and Coles (1999) argue that it is in the formation of long-term relationships, such as marriage, that we should expect matching frictions to play the strongest role. Aiyagari et al. (2000) and Fernandez et al. (2005) showed that economic models of marriage based on search frictions can successfully account for patterns of marital sorting in the US and around the world, respectively. This approach can be extended to explain cross-sectional patterns in fertility and female labor supply, as demonstrated by Caucutt et al. (2002).

In this paper we propose a search-theoretic model of marriage and births that generates long periods of adjustment to one-time demographic shocks. We then ask to what extent the model can account for the transition in post-war France. Our analysis is based on three key economic insights. First, disruption of the marriage market results in an unusually high concentration of singles with a high propensity to marry. This may explain the high marriage rates of women after the war. Second, the scarcity of men in one generation, resulting from the casualties of war, is transmitted, via the resource constraint, to future generations. This may explain the protracted nature of the transition. Third, the propensity to marry is closely linked, or may even be identical to, the propensity to have children. Thus the post-war increase in marital birth rates may be due, at least in part, to the unusual composition

[1] This pattern, of a war-time marriage bust followed by a protracted marriage boom, has been characteristic of France since at least the Napoleonic era, as Chasteland and Pressat (1962) make clear. Festy (1984) documents the effects of the Great War on birth rates in France, and Caldwell (2004) reports that these effects are not specific to France and World War I, but instead are common to many countries during various conflicts, civil wars and revolutions.

[2] France is special not just in its prolonged exposure to the war, but also in the richness of its historical vital statistics. It is the only large European country for which we have marriage and birth statistics by age of the spouses/parents before the first world war.

of the singles pool after the war.

We implement these insights in the model by assuming that fertility depends on both the age and the type of the woman. The type indexes permanent differences in the desire (or ability) of women to have children. We also assume that the value of a marriage is increasing in the number of children born to the couple. Matching is through the competitive-search mechanism: men decide which age and type of woman to court, knowing that they will get a smaller share of the value of the marriage from courting the most popular women. Thus, at an equilibrium of the marriage market, women with a higher desire (or ability) to have children marry at a higher rate.

Using a simple static example, we show that, when men are in excess supply, an increase in the quality of the pool of single women causes both male and female marriage rates to increase. This is critical because under the constant-returns-to-scale assumption that our model shares with almost all other papers in the matching literature, the marriage-rate boom cannot be explained by the presence of an unusually large singles pool. The lower post-war sex ratio, on the other hand, tends to reduce female marriage rates, which, in the full model, helps to explain why marriage rates of young women fall back to trend more quickly than the corresponding male marriage rates.

Our quantitative strategy is as follows. First, we calibrate the steady state of the model to match the age profiles of the marital status of men and women, as well as the birth-rate age-profile of married couples. Second, we compute a post-war distribution of men and women by marital status, and of married couples by already-born children. We do this by imposing that, starting from the steady state, the economy faces four years of reduced marriage, reduced birth rates, and increased male mortality, affecting all types equally by age. Third, we use this post-war distribution as the initial condition from which we compute equilibrium transition paths for marriage and birth rates. We then compare marriage and birth statistics drawn from the transition path of the model to empirical analogues drawn from French vital statistics over the 1920-1935 period.

We find that the model generates a large increase in marriage and fertility rates after the war. The shares of the marriage-rate boom explained by the model ranges from 58% to 82%, depending upon gender and age group. The model explains 59% and 79% of the increase in fertility for 20-29 and 30-39 year-old married women, respectively. The model is also consistent with the protracted nature of the transition of marriage and fertility rates. Marriage rates in the model do not converge back to their trend until 1925 or later. The persistence of excess marriage rates is particularly pronounced for young men (aged 20-29). The marital birth rate also exhibits a slow transition in the model. Finally, the model also reproduces the faster convergence of the marriage rate of women relative to that of men, as can be observed in the French data.

Our model also has implications for other cases of demographic imbalance, not just those caused by war. For instance in China, the male-female ratio by age cohort has been increasing since implementation of the One-Child policy in 1979. To date the literature on the marriage-market impact, such as Anderson and Leo (2009) have abstracted from the inter-temporal squeeze. Faced with a shortage of same-age women, we expect that these cohorts will marry younger women, which our model predicts will transmit the shortage of women to younger birth cohorts, which also face their own shortage of women. Our model could be used to ask what the impact would be, given the demographic distortion implied by 40 years of the One-Child policy, of rescinding the policy.

Our approach is complementary to the analysis of the impact of the First World War on French fertility by Vandenbroucke (2013); however his model abstracts from marriage matching and generates a baby boom after the war that is much larger than in the data. In combination with our approach, which abstracts from the incentives of married couples to have children, this suggests that marriage-market frictions delayed the response of the French population to stronger fertility incentives.

Our model is similar in spirit to that of Kennes and Knowles (2012) (KK hereafter), in that it analyzes fertility and marriage in a competitive-search framework that derives in part from the labor-market models of Moen (1997) and Peters (1991). However our model extends KK in that we allow for two-sided heterogeneity, using the methods of Shi (2002) and Shimer (2005), who extended the competitive-search framework to deal with worker heterogeneity.

Other recent economic analyses of war-related demographics includes Kvasnicka and Bethmann (2007), who analyze the effect of the second World War on out-of-wedlock births in Germany. There is also a rapidly growing literature on the impact of war on female labor supply, such as Doepke et al. (2007), who also considers the impact of labor demand on fertility, but abstracts from marriage markets. Economic models have also proved useful for understanding fertility fluctuations, as demonstrated by Jones and Schoonbroodt (2010) and Greenwood et al. (2005), but that work abstracts from the marriage-market mechanism that we stress here.

More generally, our work is also related to other marriage models that stress the role of aging and differential fecundity, such as Siow (1998) andGiolito (2010). However the focus of that work is very different, and does not deal with the impact of demographic shocks. Coles and Francesconi (2011) and Regalia and Ríos-Rull (1999)also develop models of aging in the marriage market, but their focus is on wage inequality.

In Section 2 below we review the data on marriage and births in France in the post-war era. In Section 3 we present a simple, one-period model that is useful to gain intuition about the key mechanisms at works in the dynamic model that we later use for our quantitative analysis. We present the full, dynamic model in Section 4 and our quantitative analysis in Section 5. We conclude in Section 7.

2 Empirical Analysis

In this section we break down marriage and birth rates by age to show that the effects of the war-time disruption of family life were much longer-lasting than is apparent from the aggregate statistics. In particular, we find that marriage and birth rates of the younger-non-mobilized cohorts were also disrupted, and that some of these effects are still apparent in the 1930s.

Marriage statistics are available from the French National Institute for Statistics and Economic Studies (Insee). Bunle (1954) also provides a wealth of monthly statistiques on marriages.Mitchell (1998) provides statistics for the aggregate marriage and birth rates. As far as we know, the relevant data are only available in the form of statistics, not at the individual level. This is a significant limitation, one that applies to all European countries at this time, but France is unique in that the statistics are crossed with age, whereas the UK, Italy and Germany do not provide marriage and birth rates by age before the 1930s.

Figure 1 shows aggregate marriage and birth rates in France, from 1801 to 1990. We consider the marriage rates first. It takes about 5 years after the Franco-Prussian war of 1870 for marriage rates to return to normal, 10 years after World War I, and 5 years after World War II. In contrast to the aftermath of the French invasion of Russia in 1812, where the post-war adjustment is comparatively rapid, these wars are associated with prolonged disruption due to the protracted presence of hostile forces on French soil. Turning to the birth rates, there is a noticeable trough during World War I, followed by a rebound above trend which lasted until the late 1930s. A less pronounced trough coincides with the Franco-Prussian war.[3]

In Figure 2, where we break down marriage rates by age and sex, we see that the post-war disruption is of longer duration than that suggested by the aggregate marriage rate. What is particularly intriguing in this figure is that it demonstrates that young cohorts who were not mobilized appear to also exhibit a significant disruption in marriage patterns. For example, the marriage rate of 20-29 year-old men in the late 1920s is markedly above its

[3]The baby boom in France, visible in Figure 1, started in the early 1940s.

pre-war trend, while this cohort was too young, in 1914, to have been mobilized and have its marriage decisions perturbed by the onset of the war. Figure 3 conveys a similar message for birth rates. It reveals that the marital fertility of women, for two age groups, is more disrupted than what the aggregate birth rate of Figure 1 suggested. There is a persistent shift, of about 0.05 annual births per married woman, which occurs in the birth rates of the young cohorts of women.

A stylized presentation of the qualitative feature of marriage rates after their post-war peak is:

- 20-29 year-old men: Marriage rates remain above trend durably until the mid-1930s
- 30-39 year-old men: Marriage rates revert to trend in the mid-1920s
- 20-29 year-old women: Marriage rates fell sharply in the early 1920s. They remain slightly below trend thereafter.
- 30-39 year-old women: Marriage rates revert to trend in the early 1920s. They remain on trend thereafter.

Henry (1966) shows that despite the shortage of men, the fraction of French women of the mobilized birth cohorts who ended up marrying by age 50 was surprisingly similar to that of the pre-war cohorts. These women adapted, he argues, in several different ways, but the principal one was that they selected husbands from younger birth cohorts. His results, summarized in Table 1 below, show that the increase in intercohort marriages accounts for about 52-64% of the deviation of marriage patterns in response to the war. He ignores however the impact on the women of the younger cohorts of this pre-empting by the older women of younger men. Our empirical analysis suggests that women of successive birth cohorts followed the suit of the mobilized cohort by marrying younger men, thus transmitting the shortage of men to the non-mobilized cohorts.

3 A Simple Example

A fundamental insight in our analysis is that the high marriage rates of women after the war could be explained by an unusually high concentration of singles with a high propensity to marry. We illustrate this with a comparative-statics exercise in the context of a simple model.

The economy lasts one period and is populated by two continua of singles of each sex $i \in \{H, F\}$. There are two types of women: $z_F \in \{1, 2\}$, where z_F indexes the intensity of their desire to raise children. There is only one type of man, with mass α. Let the mass of women of type z_F be $P_F(z_F)$. We normalize the total mass of women to unity, so that α equals the sex ratio.

Matching and allocation of the surplus are determined by a competitive-search mechanism, as in Shi (2002). All agents may enter the marriage market, where matches are made between agents of opposite sexes. Men pay a stochastic cost ξ to participate. We assume that ξ is identically and independently distributed with cumulative distribution function Γ, and that it is drawn at the beginning of the period, before the participation decisions of men are made. All women enter the market, where they are assigned to a sub-market z_F corresponding to their type. Participating men choose which sub-market to enter.

Let the mass of men who enter sub-market z_F be denoted $N(z_F)$. In each sub-market men are allocated randomly to the women. Following the literature, we refer to this as the "urn-ball" mechanism[4]. We define the "queue length" as $\phi(z_F) \equiv N(z_F)/P_F(z_F)$, that is the ratio of men per z_F-women in sub-market z_F. The urn-ball mechanism implies that the probability distribution $\omega_n(z_F)$ over the number n of suitors per z_F-woman is given by a standard formula:

$$\omega_n(z_F) = \frac{[\phi(z_F)]^n e^{-\phi(z_F)}}{n!}.$$

The probability that a z_F-woman has no suitors is, therefore, $\omega_0(z_F) = e^{-\phi(z_F)}$. Let $\rho(z_F)$ be the probability that a z_F-woman marries with a man:

$$\rho(z_F) = 1 - \omega_0(z_F)$$

that is, it is the probability that she has at least one suitor.

Let $x(z_F) > 0$ denote the surplus of a marriage. We assume that the output of unmatched agents, the autarky value, is zero. Women post wages $w(z_F) \leq x(z_F)$ that they will award to the man they marry. This wage is common knowledge. If a woman has more than one suitor, then she chooses among them at random. We assume that the surplus satisfies $x(1) < x(2)$, so that the surplus from a marriage with a 2-woman is larger.

[4]Burdett et al. (2001) show that this matching function can be derived as the limiting case of a strategic game of directed search.

3.1 Optimization

Let v_H denote the expected value of participating in the marriage market for a man. Because of the entry cost, there is a marginal man that is indifferent between participation and non-participation. We denote the entry cost of the marginal man by ξ^*. A man participates in the marriage market if and only if his cost is less than that of the marginal man: $\xi \leq \xi^*$. Since we assume that the cost ξ is identically and independently distributed, the fraction of men participating is $\Gamma(\xi^*)$.

The total number of marriage by z_F-women to men is $\rho(z_F) P(z_F)$. Thus, the rate at which men marry in sub-market z_F is $\rho(z_F) P(z_F) / N(z_F) = \rho(z_F) / \phi(z_F)$. It follows that, given the wages posted by women, the expected value of entering the sub-market z_F for a man, conditional on participating, is

$$E_H(z_F) = \frac{\rho(z_F)}{\phi(z_F)} w(z_F).$$

A z_F-woman posts wage offers so as to maximize her expected gain from marriage. She takes men behavior as a constraint, that is she posts wages so that participating men are indifferent, at an interior, between entering either sub-market. Thus, a woman's optimization problem writes:

$$v_F(z_F) = \max_{w(z_F)} \rho(z_F) [x(z_F) - w(z_F)] \tag{1}$$

$$s.t. \quad v_H = E_H(z_F) \tag{2}$$

3.2 Equilibrium

A matching equilibrium consists of a value for men v_H, a queue vector $\phi(z_F)$ and a cost threshold ξ^* such that:

1. Men and women are optimizing:

   (a) The queue vector solves the optimization problem of women, in (1)-(2), given the values of men.

   (b) The marginal man is indifferent between participation and non-participation: $\xi^* = v_H$.

2. Markets clear:

(a) The demand for men equals the supply: $\phi(z_F) P_F(z_F) = N(z_F)$.

(b) The allocation of men to women is feasible: $\sum_{z_F} N(z_F) \leq \alpha\Gamma(\xi^*)$.

3.3 Discussion

The first order condition of the women's optimization problem is $e^{-\phi(z_F)} x(z_F) = v_H$. This implies that, at an equilibrium where there are men entering each sub-market, the following indifference condition must hold for men $e^{-\phi(1)} x(1) = e^{-\phi(2)} x(2)$, or

$$\phi(2) = \phi(1) + \ln\left(\frac{x(2)}{x(1)}\right). \tag{3}$$

The market-clearing conditions imply :

$$\phi(1) + P_F(2) \ln\left(\frac{x(2)}{x(1)}\right) = \alpha\Gamma(\xi^*). \tag{4}$$

We use these equations to derive three useful properties of the equilibrium: (1) type-2 women marry at a higher rate than type-1 women; (2) the participation rate of men is increasing in the fraction of 2-women; (3) the marriage rates of each type of woman is increasing in men's participation.

To derive the first property, we note that Equation (3) implies that $\phi(2)$, the queue length for type-2 women, is larger than $\phi(1)$, the queue length for type-1 women. Since the marriage rate of a z_F-women is $\rho(z_F) = 1 - e^{-\phi(z_F)}$, it follows that the marriage rate of type-2 women is larger than that of type-1 women.

To derive the second property, we note that the participation constraint of men combined with the first order condition of the women's optimization problem imply $\phi(z_F) = \ln(x(z_F)/\xi^*)$. Combining with Equation (4) implies that, in equilibrium, the entry cost of the marginal man satisfies:

$$\Gamma(\xi^*) + \frac{1}{\alpha}\ln(\xi^*) = \frac{1}{\alpha}\left[\ln(x(1)) + P_F(2)\ln\left(\frac{x(2)}{x(1)}\right)\right].$$

It follows that an increase in the proportion of type-2 women, $P_F(2)$, yields an increase in the cost threshold ξ^*, i.e. an increase in the participation of men.

To derive the third property we note that Equation (4) implies that the queue length $\phi(1)$ is increasing in men's participation. Since the marriage rate of 1-women is $\rho(1) = 1 - e^{-\phi(1)}$, it follows that the marriage rate of 1-women is increasing in men's participation. Equation

(3) implies that $\phi(2)$ is increasing in $\phi(1)$, hence the marriage rate of type 2 women is also increasing in men's participation.

The average marriage rate of women is the weighted sum of the type-specific marriage rates:

$$P_F(1)\left(1-e^{-\phi(1)}\right)+P_F(2)\left(1-e^{-\phi(2)}\right)$$

What is the effect on female marriage rates of increasing the proportion of type-2 women? There are three effects. First, there is a direct effect that transpires through Equation (4): as the proportion of type-2 women increases, the queue length $\phi(1)$ decreases. Since $\phi(2)$ is increasing in $\phi(1)$, it decreases too. Thus, the first effect of increasing $P_F(2)$ is a reduction in marriage rates of each type of woman. Second, there is an indirect equilibrium effect that causes queue lengths to increase. As we have shown, an increase in the proportion of type-2 women yields an increase in men participation which, in turns, increases the marriage rate of each type of woman. Finally, there is a composition effect. Since type-2 women marry at a higher rate, increasing their proportion raises the average marriage rate of women. In the next section, we consider a numerical example where the dominating effect is the last one, implying that the average rate of marriage of women increase when the proportion of type-2 women increases.

3.4 Comparative Statics

What happens to the equilibrium marriage rates as the fraction of high-surplus women in the singles pool increases? This is shown in Figure 5. We assume a log-normal distribution for the participation cost. The parameters used for the example are shown in Table 2.

The top-left panel of Figure 5 shows that as $P_F(2)$ varies from 0 to 1, the marriage rates decline because the queue lengths decline. This is despite the fact that the cost threshold increases, as we see in the top-right panel. Nevertheless, the female marriage rate increases overall from about 20% to about 40%, because the type-2 marry at a higher rate. The male marriage rates also increase, at a faster rate than the female, because the male participation rate has increased.

We demonstrate the impact of composition of the female population on fertility by assuming that type-2 women have a higher birth rate (0.5) than the type-1 (0.25) when married. The bottom-right panel of Figure 5 shows the birth rate rising from 7% to 13% as the fraction of type-2 women increases from 0 to 1. The birth-rate increase is due to both the direct effect of the increase in high-fertility females and the increasing overall marriage rate.

The participation cost for men is the aspect of the model that is critical for the composition of the female population to impact marriage rates. Without it, all men always participate in the marriage market. Consider a world where all the women are the same; if we increase the surplus a woman generates in marriage, there will be no impact on marriage rates if all men always participate, because the queue length will be constant. Therefore an increase in female "quality" is not sufficient to generate an increase in marriage rates.

4 The Dynamic Model

In the example model, changes in composition of the female population were shown to generate significant changes in marriage and birth rates. In this section, we extend the analysis to a dynamic model so that we can think of the impact on a succession of birth cohorts. The male type of the static example will correspond to age, and the female type will correspond to age and fertility propensity. As females age, their fecundity will decline, reducing the marriage surplus, but as males age, we will assume the marital surplus increases.

4.1 Demography

Time is discrete. There is an infinite succession of periods, and a population composed of men and women. There are two types of women: $z \in \mathcal{Z} = \{1, 2\}$, where z indexes the intensity of their desire to raise children. There is only one type of man.

Over time, individuals become differentiated along two dimensions. First, their marital status: they can be either married or single, designated by M and S. Second, individuals transition stochastically through three "stages," $a \in \mathcal{A} = \{1, 2, 3\}$, as they age. Stages of life serve two purposes in our model. First, they matter for the determination of an individual's utility as a single. Second, and for women only, stages of life affect their ability to have children once they are married. Married women in the first stage of their lives are the most "fertile," while married women in the third stage are "sterile." The probability that an individual in stage $a < 3$ transitions into stage $a+1$ is denoted by δ_a. Stage 3 is an absorbing state. We assume that each individual starts life as a single in stage 1, and that marriage is an absorbing state. Each period, there is a flow $\chi_{H,t}$ of new single men in the first stage of their lives and a flow $\chi_{F,t}(z)$ of new single women of type z in the first stage of their lives.

The state of an unmarried man is given by the stage of life he is currently in. We use $s_H \in \mathcal{A}$ to refer to it. The state of an unmarried woman is given by her type and the stage of life she is currently in: $s_F \in \mathcal{Z} \times \mathcal{A}$. Married couples are identified by the wife's state and the number of children already born, i.e. a pair $(s_F, k) \in \{\mathcal{Z} \times \mathcal{A}\} \times \{0, 1, \ldots, K\}$ where K is the maximum number of children a couple can have. We assume that married people can have one child per period and single individual cannot have children. The probability that a married couple (s_F, k) has a child in a given period is denoted by $f(s_F, k)$. Our approach of fertility is complementary to that of Vandenbroucke (2013), who models the incentives of married couples to have children; we focus instead on the incentives for marriage, as implied by expected future fertility.

4.2 Marriages Value and Autarky

Single individuals produce utility $y_i^S(s_i)$, where $i \in \{H, F\}$. We assume that for men in the second stage of their lives, the utility of remaining single is lower than for men in the first stage: $y_H^S(2) < y_H^S(1)$. Marriages produce utility $y^M(s_F, k) > 0$ each period, which is perfectly transferable between spouses. Married people like children, that is $y^M(s_F, k+1) > y^M(s_F, k)$. Furthermore, the utility gain from each child is assumed to be increasing in the woman's type z. Thus, for any $a \in \mathcal{A}$, if $s'_F = \{2, a\}$ and $s_F = \{1, a\}$, then $y^M(s'_F, k+1) - y^M(s'_F, k) > y^M(s_F, k+1) - y^M(s_F, k)$. As children can only be produced in marriage, this means that it is more important for high-z women to marry early.

Let $Y(s_F, k)$ denote the value of a marriage in state (s_F, k). Since sterility, i.e. $s_F = \{z, 3\}$, is an absorbing state, the value of a sterile marriage with k children equals the present discounted sum of the utility flow of remaining in the marriage forever:

$$Y(\{z, 3\}, k) = \frac{y^M(\{z, 3\}, k)}{1 - \beta}$$

where β is the discount factor between periods. Since couples with K children are effectively sterile, we can also denote their value by the present discounted sum of the utility flow of remaining in the marriage forever:

$$Y(s_F, K) = \frac{y^M(s_F, K)}{1 - \beta}.$$

Using these expressions we can derive the value of any (s_F, k) marriage where $k < K$ and $a < 3$ as

$$\begin{aligned} Y(s_F, k) = {} & (1 - f(s_F, k)) \left[y^M(s_F, k) + \beta\left[(1 - \delta_a) Y(s_F, k) + \delta_a Y(s_F + 1, k)\right]\right] \\ & + f(s_F, k) \left[y^M(s_F, k+1) + \beta\left[(1 - \delta_a) Y(s_F, k+1) + \delta_a Y(s_F + 1, k+1)\right]\right] \end{aligned}$$

where we used the convention that $s_F + 1$ means $\{z, a+1\}$ whenever $s_F = \{z, a\}$. Consider now an agent who *never* participates in a marriage market. Let the value of this be $A_i(s_i)$, $i \in \{H, F\}$, where $s_H(3) = y_H^S(3)/(1-\beta)$, and $s_F(\{z,3\}) = y_F^S(\{z,3\})/(1-\beta)$ and

$$A_i(s_i) = y_i^S(s_i) + \beta\left[(1-\delta)A_i(s_i) + \delta_a A_i(s_i+1)\right]$$

and where, again, we used the convention that $s_F + 1$ means $\{z, a+1\}$ whenever $s_F = \{z, a\}$.

4.3 Matching and Marriage Rates

The matching mechanism involves directed search, in the spirit of Shi (2002) and Shimer (2005). Specifically men direct their search for a spouse toward sub-markets of women of type s_F. In sub-market s_F, a woman posts a wage offer $w_t(s_F, s_H)$ that she will pay to her husband. The wage offer is common knowledge and women are able to commit to their offers. Matching in a given sub-market occurs through the random assignment of men to women, using the urn-ball mechanism, as described in section 3.

4.3.1 Matching

At the beginning of period t the population of singles of sex i and state s_i is denoted by $P_{i,t}(s_i)$. Men pay a cost ξ to enter the marriage market. We assume that ξ is iid with CDF Γ, and that it is drawn at the beginning of each period, before the participation decision is made. All women participate in the market, where they are assigned to a sub-market s_F corresponding to their state. Men choose which sub-market to enter. Let the mass of s_H-men who enter sub-market s_F be denoted $N_t(s_F, s_H)$.

The "queue length" or "market-tightness" in sub-market s_F is

$$\phi_t(s_F, s_H) \equiv \frac{N_t(s_F, s_H)}{P_{F,t}(s_F)}.$$

The urn-ball mechanism implies that the probability distribution $\omega_{n,t}(s_F, s_H)$ over the number n of s_H-suitors per s_F-woman is given by:

$$\omega_{n,t}(s_F, s_H) = \frac{\left[\phi_t(s_F, s_H)\right]^n e^{-\phi_t(s_F, s_H)}}{n!}.$$

The probability that there are no s_H-suitors for an s_F-woman is $\omega_{0,t}(s_F, s_H) = e^{-\phi_t(s_F, s_H)}$. Let $\rho_t(s_F, s_H)$ be the probability that an s_F-woman marries with an s_H-man. A woman matches with a 2-man whenever at least one suitor is a 2-man. Thus

$$\rho_t(s_F, 2) = \sigma[1 - \omega_{0,t}(s_F, 2)], \tag{5}$$

where σ is the probability of marriage conditional on a match. A woman matches with a 1-man whenever she has at least one suitor of this type and no type-2 suitor:

$$\rho_t(s_F, 1) = \sigma\omega_{0,t}(s_F, 2)(1 - \omega_{0,t}(s_F, 1)). \tag{6}$$

4.3.2 Marriage Rates

Women The total number of marriages by s_F-women is $[\rho_t(s_F, 1) + \rho_t(s_F, 2)] P_{F,t}(s_F)$. Thus, the marriage rate of these women is $\pi^F_{F,t}(s_F) = \rho_t(s_F, 1) + \rho_t(s_F, 2)$, which is also

$$\pi_{F,t}(s_F) = \sigma[1 - \omega_{0,t}(s_F, 2)\omega_{0,t}(s_F, 1)], \tag{7}$$

i.e., the probability that they have at least one suitor.

Men The number of marriages between s_H-men and s_F-women is $\rho_t(s_F, s_H) P_{F,t}(s_F)$. This implies that the rate at which s_H-men marry in market s_F is

$$\frac{\rho_t(s_F, s_H) P_{F,t}(s_F)}{N_t(s_F, s_H)} = \frac{\rho_t(s_F, s_H)}{\phi_t(s_F, s_H)}.$$

The marriage rate for s_H-men can, thus, be written as

$$\pi_{H,t}(s_H) = \sum_{s_F} n_t(s_F, s_H) \frac{\rho_t(s_F, s_H)}{\phi_t(s_F, s_H)} \tag{8}$$

where

$$n_t(s_F, s_H) = \frac{N_t(s_F, s_H)}{\sum_{s_F} N_t(s_F, s_H)}$$

is the proportion of s_H-men in market s_F. Note that $\pi_{H,t}(s_H)$ is the marriage rate of men, conditional on participating. The unconditional marriage rate of single men is

$$\pi_{H,t}(s_H)\mu_t(s_H)$$

where $\mu_t(s_H)$ is the probability of participation, i.e. the fraction of s_H-men participating in the marriage market.

4.3.3 Laws of Motion

Given marriage and participation rates, the laws of motion for the population of single $a_H = 1-$men is

$$P_{H,t+1}(1) = (1 - \delta_1)[1 - \pi_{H,t}(1)\mu_t(1)]P_{H,t}(1) + \chi_H, \tag{9}$$

that is, the number single, $a_H = 1-$men at date $t+1$ comes from the date t single in stage 1 that did not transition into stage 2 and did not get married, and from the flow of new single of age 1. For $a_H = 2$-men we have

$$P_{H,t+1}(2) = (1 - \delta_2)[1 - \pi_{H,t}(2)\mu_t(2)]P_{H,t}(2) + \delta_1[1 - \pi_{H,t}(1)\mu_t(1)]P_{H,t}(1). \tag{10}$$

Thus, the number single, $a_H = 2-$men at date $t+1$ comes from the date t single in stage 2 that did not transition into stage 3 and did not get married, and from single in stage 1 at date t that transitioned into sage 2 but did not get married.

For women, these laws can be written as

$$P_{F,t+1}(\{z,1\}) = (1 - \delta_1)[1 - \pi_{F,t}(\{z,1\})]P_{F,t}(\{z,1\}) + \chi_F(z) \tag{11}$$

for $\{z, 1\}$-women, and

$$P_{F,t+1}(\{z,2\}) = (1 - \delta_2)[1 - \pi_{F,t}(\{z,2\})]P_{F,t}(\{z,2\}) + \delta_1[1 - \pi_{F,t}(\{z,1\})]P_{F,t}(\{z,1\}) \tag{12}$$

for $\{z, 2\}$-women.

4.4 Value Functions

4.4.1 Men

Let $V_{H,t}(s_H)$ denote the value of a man who has decided to participate in the market. Let $R_{H,t}(s_H)$ denote the value of remaining single and $v_{H,t}(s_H)$ denote the expected gain from marrying during the period. We have

$$V_{H,t}(s_H) = R_{H,t}(s_H) + v_{H,t}(s_H).$$

To define the value of remaining single for men we proceed as follows. First, we note that s_H-men enter the market if and only if $\xi \leq v_{H,t}(s_H)$, that is if and only if their expected gain from marrying conditional on participating is at least as large as the cost of participating. We use the notation $\xi_t^*(s_H) = v_{H,t}(s_H)$ to denote the marginal s_H-man that is exactly indifferent between participation and non-participation. Thus, the probability that an s_H-man participates, $\mu_t(s_H)$, is

$$\mu_t(s_H) = \Gamma(\xi_t^*(s_H)). \tag{13}$$

Since ξ is iid, $\mu_t(s_H)$ is also the proportion of s_H-men participating in the marriage market at date t. Before the realization of the cost, the ex-ante value of period t for an s_H-man is then

$$W_{H,t}(s_H) = (1 - \mu_t(s_H)) R_{H,t}(s_H) + \mu_t(s_H) E[V_{H,t}(s_H) - \xi \,|\xi \leq \xi_t^*(s_H)],$$

which is also

$$W_{H,t}(s_H) = (1 - \mu_t(s_H)) R_{H,t}(s_H) + \mu_t(s_H)[V_{H,t}(s_H) - \zeta_t(s_H)],$$

where $\zeta_t(s_H) \equiv E[\xi \,|\xi \leq \xi_t^*(s_H)]$ is the expected cost conditional on participating. Then, the value of remaining single for a man is

$$R_{H,t}(s_H) = y_H^S(s_H) + \beta[(1 - \delta_{s_H}) W_{H,t+1}(s_H) + \delta_{s_H} W_{H,t+1}(s_H + 1)]. \tag{14}$$

Note that $R_{H,t}(3) = W_{H,t}(3) = A_H(3)$.

4.4.2 Women

Let $V_{F,t}(s_F)$ denote the value of a woman at the beginning of period t. Let $R_{F,t}(s_F)$ denote the value of remaining single and $v_{F,t}(s_F)$ denote the expected gain from marrying during the period. We have

$$V_{F,t}(s_F) = R_{F,t}(s_F) + v_{F,t}(s_F),$$

and

$$R_{F,t}(s_F) = y_F^S(s_F) + \beta[(1 - \delta_{s_F}) V_{F,t+1}(s_F) + \delta_{s_F} V_{F,t+1}(s_F + 1)]. \tag{15}$$

A woman chooses to post wages so as to maximize her expected gain from marrying, subject to the constraint that participating men must receive their expected values. Thus $v_{F,t}(s_F)$

is defined by the following optimization problem

$$v_{F,t}(s_F) = \max_{w_t(s_F,s_H)} \sum_{s_H} \rho_t(s_F,s_H)\left[x_t(s_F,s_H) - w_t(s_F,s_H)\right] \tag{16}$$

$$s.t. \qquad v_{H,t}(s_H) = \frac{\rho_t(s_F,s_H)}{\phi_t(s_F,s_H)} w_t(s_F,s_H), \tag{17}$$

where the surplus $x_t(s_F,s_H)$ is defined by the output of a marriage, net of the reservation values of the husband and the wife:

$$x_t(s_F,s_H) = Y(s_F,0) - R_{H,t}(s_H) - R_{F,t}(s_F). \tag{18}$$

4.5 Equilibrium

A recursive equilibrium of the model is a sequence of queue vectors $\{\phi_t(s_F,s_H)\}$, cost thresholds $\{\xi_t^*(s_H)\}$, expected values $\{v_{H,t}(s_H), v_{F,t}(s_F)\}$ for men and women, respectively, reservation values $\{R_{H,t}(s_H), R_{F,t}(s_F)\}$, for men and women, and populations of single men and women $\{P_{H,t}(s_H), P_{F,t}(s_F)\}$ such that at each date t:

1. Men and women are optimizing:
   (a) The reservation value of men satisfies Equation (14).
   (b) The reservation value of women satisfies Equation (15).
   (c) Women solve problem (16)-(17).
   (d) The marginal man is indifferent between participation and non-participation: $\xi_t^*(s_H) = v_{H,t}(s_H)$.
2. Markets clear:
   (a) The demand for men equals the supply: $\phi_t(s_F,s_H) P_{F_t}(s_F) = N_t(s_F,s_H)$.
   (b) The allocation of men to women is feasible: $\sum_{s_F} N_t(s_F,s_H) \leq \Gamma(\xi_t^*(s_H)) P_{H,t}(s_H)$.
3. The population of singles follows the laws of motion in Equations (9)-(12).

5 Quantitative Analysis

5.1 Calibration

We use the following functional forms for the output of a marriage

$$y^M(\{z, a_F\}, k) = \alpha_0 + \alpha_1 \ln(1+k),$$

where α_0 and α_1 are non-negative. Note that we assume that the output of a marriage depends only upon the number of children. For fertility we assume

$$f(\{z, a_F\}, k) = \frac{f_z}{f_z + f_a f_k},$$

where f_z, f_a, and f_k are non-negative.

We choose parameter values for the model in two ways. Some parameters can be set *a priori* on the basis of previous literature. An annual discount rate of 4% is standard in the macroeconomic literature because it generates an average rate of return that matches the risk-free interest rate. Similarly the expected duration of fecundity, as of age 18, is 22 years according to Trussell and Wilson (1985). We set the transition probability $\delta = 0.1$ so as to generate 20 years of fecundity on average. The inflow rate of women is set to $\chi = 0.1$ so as to keep the population size constant.

The remaining parameter values are chosen so that the model generates steady-state age profiles for marriage and births that match those in the data. The empirical counterparts consists of averages for French pre-war vital statistics, as described in the empirical analysis of section 2.

This strategy leaves one important parameter unidentified: the variance σ_ξ of the participation cost, which we set to 0.1. The smaller the variance, the greater the response of male participation to the quality of the single females pool. In future versions we plan to identify this parameter using the results for other wars where the extent of marital disruption differed from the First World War.

Figure 6 show the profiles of married men and women, by age, as well as birth rates compared with their empirical counterpart. The calibrated values of the model parameters are shown in Table 3.

5.2 The Main Experiment

5.2.1 The war

The first step in our experiment is to build the post-war distribution of the population. We proceed as follows. We start from the steady state of our model and assume that, for 4 years, (i) marriages rates are divided by two; (ii) 4% of men die each year; (iii) the fertility of married couples is divided by two. Under these assumptions we derive the post-war composition of the population, which serves as the initial condition from which we compute a transition path.

Our assumption that marriage rates are divided by two during the war is motivated by the data of Figure 2 which show that early in the war marriages rates fell by about 50%. Our assumption that 4% of men die each year is motivated by data on military casualties during the war –see Huber (1931). Of the 8.5 million men mobilized. 16% died, i.e. about 4% per year for four years. Finally, our assumption that the fertility of married couples was divided by two is also motivated by data on marital fertility during the war.

5.2.2 The results

Figure 7 shows the results of the experiment. It displays marriage rates relative to their steady state in the model, and marriage rates relative to their pre-war trend in the data. We start with a qualitative discussion of these results. The model generates an increase in marriage rates for men and women, and for the two age groups that we consider. For 20-29 year-old men, the data show that marriage rates remain above trend (that is 1 in Figure 7) until the mid-1930s. The model generate a faster-than-observed return to trend, which is almost completed by 1925. For 30-39 year-old men, the pace of the return to trend is similar in the model and the data. That is, by 1925 most of the convergence has occurred and marriage rates remain close to trend.

The marriage rate of 20-29 year-old women declines sharply from its peak in the early 1920s. It then remains slightly below trend. The model is consistent both with the initial decline and its timing, and with the lower-than-trend rates of the subsequent years. For 30-39 year old women, marriage rate revert to trend in the early 1920s, and remain on trend afterward. The model exhibits a slower decline that is completed in the late 1920s. It remains close to trend afterward.

Table 4 summarizes this discussion quantitatively. The table shows the ratio of the model and data lines of Figure 7. Our model predicts, 69% of the deviation from trend of the marriage rates of 20-29 year-old men in 1920. For 30-39 year-old men it predicts 58% of the deviation from trend. For young and old women these figures are 82 and 71%, respectively. In the years following the 1920 peak, the deviation from trend generated by our model remain in the neighborhood of the actual deviation. For 20-29 year old men, in 1930, for instance, the model predicts 94% of the actual deviation.

To understand these results we note that our model implies that the queue lengths in the $a_F = 1$-markets start above their steady state values immediately after the war, but then fell and remain below their steady state value, exhibiting a slow convergence back to steady state. This is an indication that young women do not attract men after the war as they used to. This is, in part, due to the competition of older women, that is $a_F = 2$-women, who face longer queues of young men after the war. Indeed, our results show that queues of young ($a_H = 1$) men in the markets for older ($a_F = 2$) women are above their steady state values after the war.

5.3 Decomposition

5.3.1 The role of low marriages during the war

5.3.2 The role of low births during the war

5.3.3 The role of high male mortality during the war

6 Other Applications

7 Conclusions

This paper explains how cohort-specific demographic shocks are transmitted through the marriage market to successive birth cohorts. We used the example of France after the First World War to illustrate the dynamics imposed by two events which we take to be exogenous: the disruption of family life during the war and the post-war shortage of single men in the mobilized cohorts due to combat-related mortality.

By breaking down the data by age we were able to show that the disruption of marriage markets was much longer lasting than apparent from the aggregate data, and that the non-mobilized younger cohorts were also affected by these events, as evidenced by both the male marriage rates and the marital fertility rates remaining considerable higher than trend into the 1930s.

The premise of our theoretical model was that women differ in their birth propensities, and hence in their gains from marriage. While a small economic literature on differential fecundity has developed models in which female aging affects fecundity, this is to our knowledge the first equilibrium model of marriage that explicitly accounts for differences in preferences over births.

We first used a simplified example of the model to show how improved composition of the female singles pool leads to higher marriage rates; this argument relied on some men preferring sometimes not to participate in the marriage market. We then calibrated the model's steady state to match the pre-war age profiles of marital status and births; this generated a remarkably good fit, despite the simple demographic structure of the model.

Our main result was that when we shut down family life for years, in imitation of the effects of the war, the transition dynamics exhibit features similar to those in the French data. We see marriage rates peak immediately after the war and then eventually decline, and birth rates remain above trend for many years.

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Female Birth Cohort	Predicted Spinster Rate	Marriage with Foreigners	Marriages with widowers and divorces	Reduced rate of Bachelor-hood	Inter-cohort marriages	Total Adjustment	Share of IC Marriages
1891-1895	196	2	14.6	14	34.4	65	53%
1896-1900	226	15	10.2	13.6	68.2	107	64%
1901-1905	164	12	1.4	10.2	34.4	58	59%
1906-1910	115	5	0.7	5.8	12.5	24	52%

Table 1: *Predicted Spinster Rate per 1000 women and demographic adjustments*. Source: Henry(1966)

Parameter	Value
marriage surplus z=1 a=1	5.0
marriage surplus z=2 a=1	6.9
marriage surplus z=1,a=2	5.1
marriage surplus z=2,a=2	7.0
parameter 1 of the cost distribution function	1.4
parameter 2 of the cost distribution function	0.2
SexRatio	0.9
Low Fertility rate	0.3
High Fertility rate	0.5

Table 2: *Parameter Values for Simple Model*

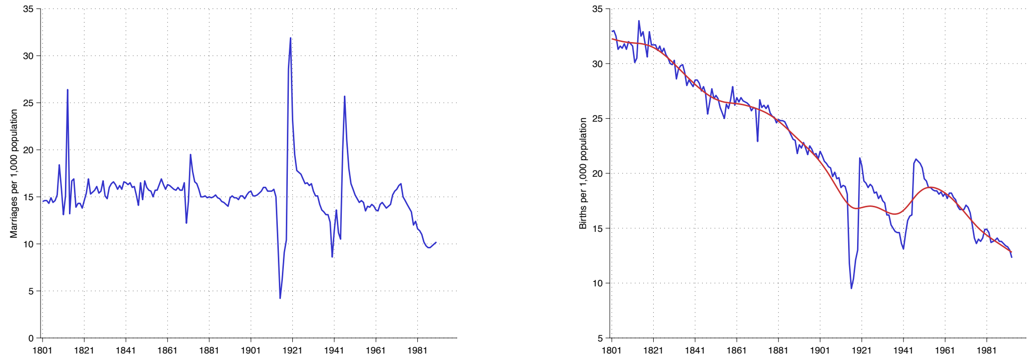
Name	Value	Role
β	0.96	Discount factor
δ	0.1	Transition through stages
$y_S^H(a_H)$	[10.0,2.0,0.0]	Output of single men
$y_S^F(a_F)$	[3.0,0.1,0.0]	Output of single women
α	5.9	Output of marriage, intercept
α	2	Output of marriage, slope
f_a	[5.0,2.3]	Fertility function, effect of a_{F}
f_k	[0.09,0.15,0.3]	Fertility function, effect of k
f_z	[0.15,0.4]	Fertility function, effect of z
χ	0.2	Inflow of men
χ	[0.1,0.1]	Inflow of women
σ	0.5	Probability of marriage given match
μ	0.5	Mean of entry cost
σ_χ	0.1	Standard deviation of entry cost

Table 3: *Parameter Values for Benchmark Model*

Sample	1913	1920	1925	1930	1935
Marriage rates relative to trend: model/data					
Men, 20-29	1	0.69	0.8	0.94	1.05
Men, 30-39	1	0.58	0.84	0.89	1.09
Women, 20-29	1	0.82	0.95	0.99	1.05
Women, 30-39	1	0.71	0.94	0.93	1.02
Annual birth rates per married woman: model/data					
Women, 20-29	1	0.79	1.16	1.15	1.26
Women, 30-39	1	0.59	0.76	0.82	0.87

Table 4: *The post-war transition, in the model and in the data*.

Figure 1: Aggregate marriage and birth rates, France 1800-1990

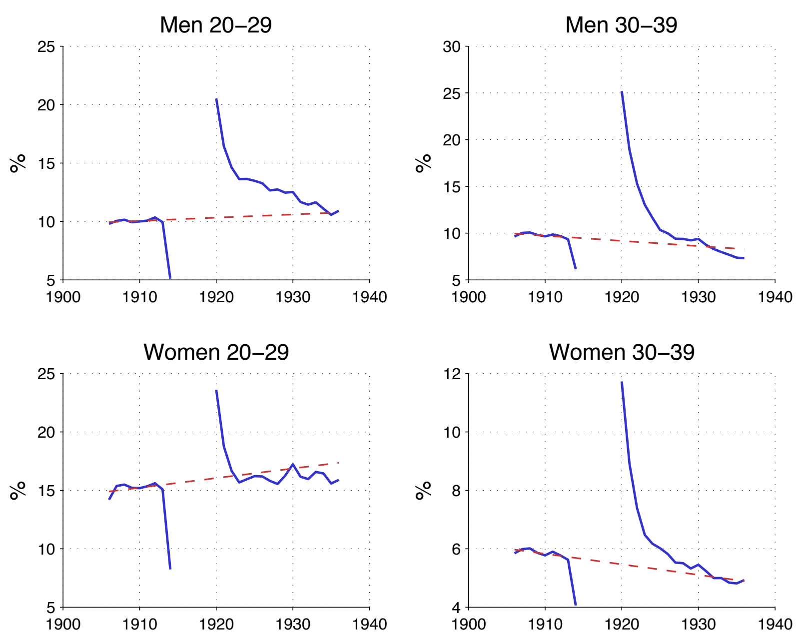


A - Marriage rates

B - Birth rates

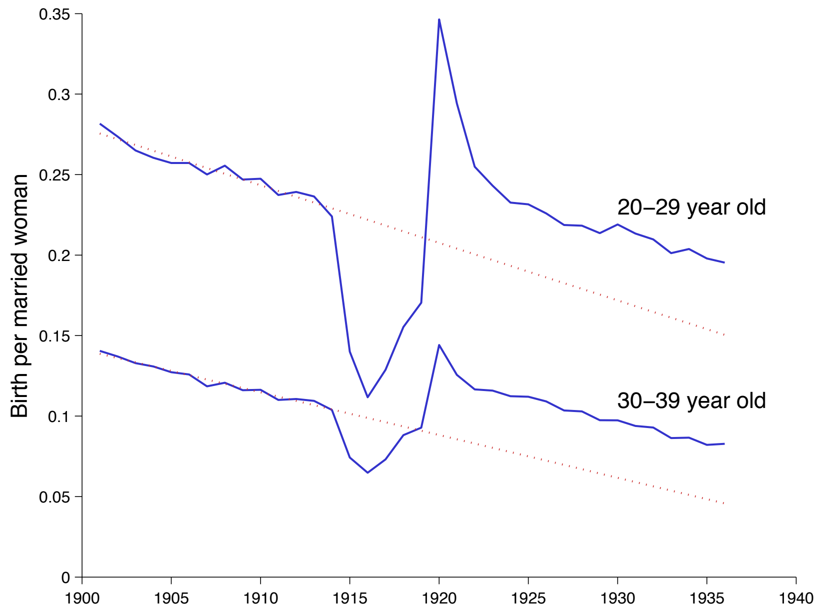
Note: The source of data is Mitchell (1998).

Figure 2: Marriage rates



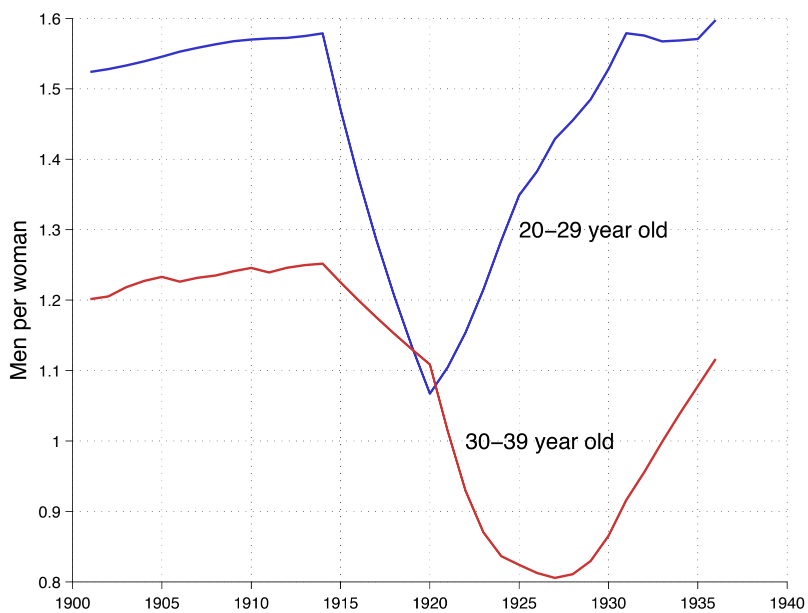
Note: The source of data is Bunle (1954), and Insee, etat civil et recensement de population. The dotted line represent the pre-war trend.

Figure 3: Birth per married women



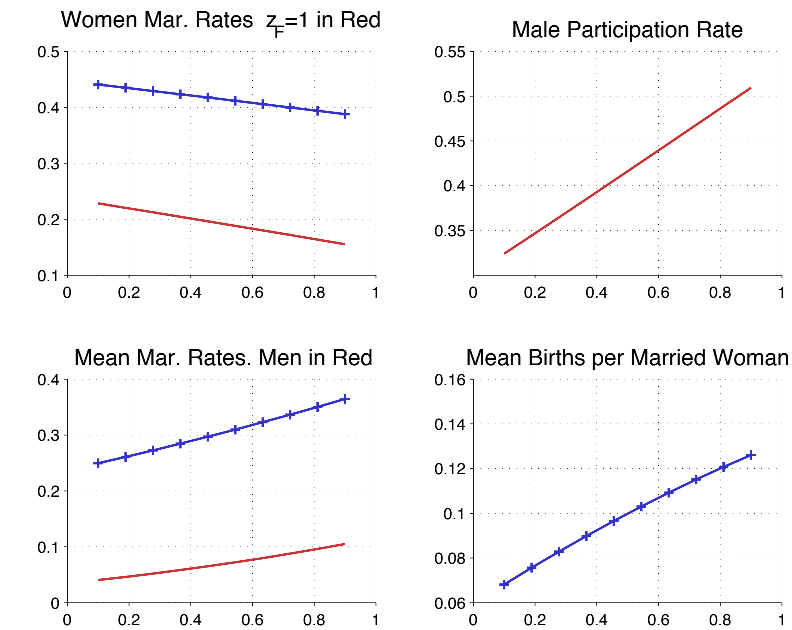
Note: The source of data is Insee, etat civil et recensement de population.

Figure 4: Single men per Single woman



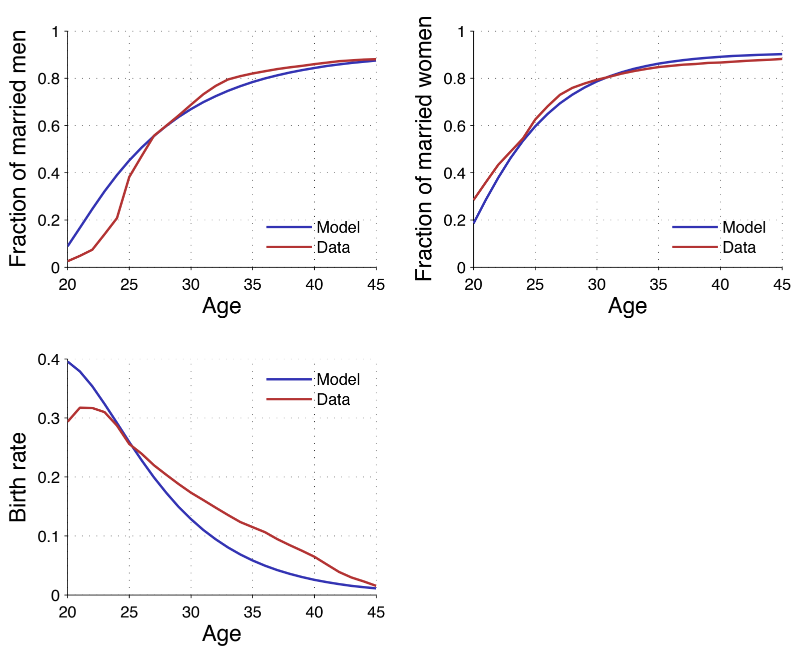
Note: The source of data is Insee, etat civil et recensement de population.

Figure 5: Example: Comparative Statics



Note: the horizontal axis represents fraction of females of type $z_F = 2$.

Figure 6: Fraction of married men and women, and birth rates in steady state of benchmark model and data



Note: The source of data is Insee, etat civil et recensement de population. The pre-war data are averages for the years 1901-1913.

Figure 7: Marriage rates relative to pre-war trend, model and data

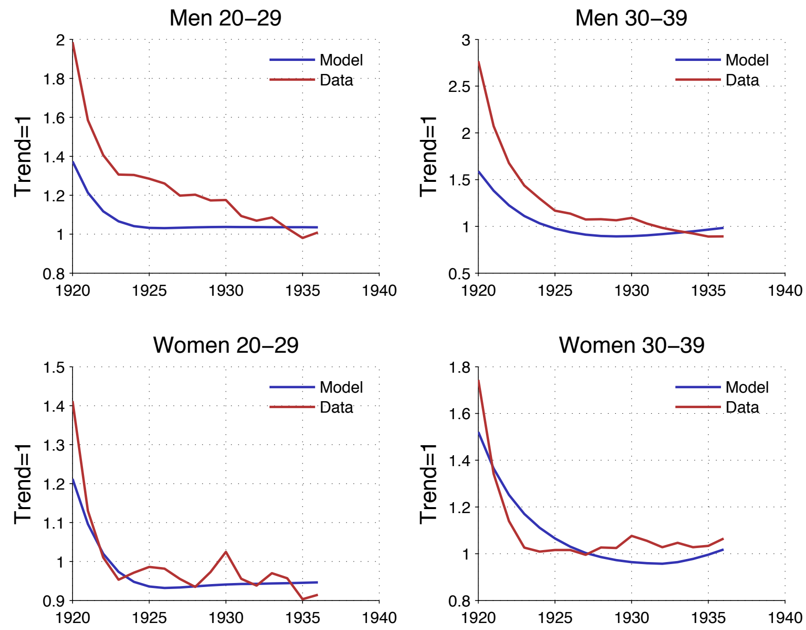
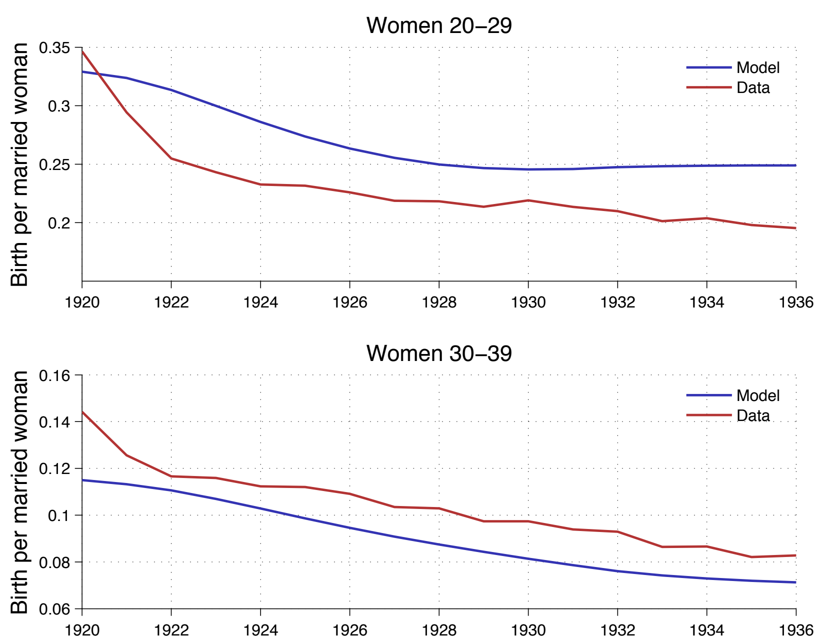


Figure 8: Birth per married women, model and data



A The Static Equilibrium of the Dynamic Model

We solve the static equilibrium using a Newton-based algorithm augmented to handle the possibility of corner solutions, i.e. when some markets are inactive. The computations are simplified by the following considerations. First, recall that

$$\begin{aligned}
\rho(s_F,1) &= \sigma e^{-\phi(s_F,2)}\left(1-e^{-\phi(s_F,1)}\right) \\
\rho(s_F,2) &= \sigma\left(1-e^{-\phi(s_F,2)}\right)
\end{aligned}$$

so that

$$\begin{aligned}
\partial\rho(s_F,1)/\partial\phi(s_F,1) &= \sigma e^{-\phi(s_F,2)}e^{-\phi(s_F,1)} \\
\partial\rho(s_F,1)/\partial\phi(s_F,2) &= -\sigma e^{-\phi(s_F,2)}\left(1-e^{-\phi(s_F,1)}\right) \\
\partial\rho(s_F,2)/\partial\phi(s_F,1) &= 0 \\
\partial\rho(s_F,2)/\partial\phi(s_F,2) &= \sigma e^{-\phi(s_F,2)}
\end{aligned}$$

The optimization problem of a woman writes, after substituting out $w(s_F,s_H)$ using the constraint,

$$\max_{\phi(s_F,s_H)} \rho(s_F,1)\,x(s_F,1)-\phi(s_F,1)\,v_H(1)+\rho(s_F,2)\,x(s_F,2)-\phi(s_F,2)\,v_H(2)$$

At an interior, the first order conditions of this problem are:

$$\begin{aligned}
\phi(s_F,1) &: \; 0=\sigma e^{-\phi(s_F,2)}e^{-\phi(s_F,1)}x(s_F,1)-v_H(1) \\
\phi(s_F,2) &: \; 0=-\sigma e^{-\phi(s_F,2)}\left(1-e^{-\phi(s_F,1)}\right)x(s_F,1)+\sigma e^{-\phi(s_F,2)}x(s_F,2)-v_H(2).
\end{aligned}$$

$$-\sigma e^{-\phi(s_F,2)}\left(1-e^{-\phi(s_F,1)}\right)x(s_F,1)+\sigma e^{-\phi(s_F,2)}x(s_F,2)=v_H(2)$$

It is convenient to re-write these equations as

$$\begin{aligned}
\phi(s_F,1) &: \; 0=-\phi(s_F,2)-\phi(s_F,1)-\ln\left[\frac{v_H(1)/\sigma}{x(s_F,1)}\right] \\
\phi(s_F,2) &: \; 0=-\phi(s_F,2)-\ln\left[\frac{v_H(2)/\sigma}{x(s_F,2)-\left(1-e^{-\phi(s_F,1)}\right)x(s_F,1)}\right].
\end{aligned}$$

We can now express the static equilibrium as a system of 10 equations in 10 unknown variables. The unknown are 8 queue lengths, ϕ, and 2 value functions for men, v_H. The equations are 4 first order conditions for wage posting toward 1-men, four first order conditions toward

2-men, and 2 resource constraints.

The first set of first order conditions is

$$\begin{aligned}
\phi\left(\left\{1,1\right\},1\right) &: \quad 0=-\phi\left(\left\{1,1\right\},2\right)-\phi\left(\left\{1,1\right\},1\right)-\ln\left[\frac{v_H\left(1\right)/\sigma}{x\left(\left\{1,1\right\},1\right)}\right] \\
\phi\left(\left\{1,2\right\},1\right) &: \quad 0=-\phi\left(\left\{1,2\right\},2\right)-\phi\left(\left\{1,2\right\},1\right)-\ln\left[\frac{v_H\left(1\right)/\sigma}{x\left(\left\{1,2\right\},1\right)}\right] \\
\phi\left(\left\{2,1\right\},1\right) &: \quad 0=-\phi\left(\left\{2,1\right\},2\right)-\phi\left(\left\{2,1\right\},1\right)-\ln\left[\frac{v_H\left(1\right)/\sigma}{x\left(\left\{2,1\right\},1\right)}\right] \\
\phi\left(\left\{2,2\right\},1\right) &: \quad 0=-\phi\left(\left\{2,2\right\},2\right)-\phi\left(\left\{2,2\right\},1\right)-\ln\left[\frac{v_H\left(1\right)/\sigma}{x\left(\left\{2,2\right\},1\right)}\right]
\end{aligned}$$

The second set of conditions is

$$\begin{aligned}
\phi\left(\left\{1,1\right\},2\right) &: \quad 0=-\phi\left(\left\{1,1\right\},2\right)-\ln\left[\frac{v_H\left(2\right)/\sigma}{x\left(\left\{1,1\right\},2\right)-\left(1-e^{-\phi\left(\left\{1,1\right\},1\right)}\right)x\left(\left\{1,1\right\},1\right)}\right] \\
\phi\left(\left\{1,2\right\},2\right) &: \quad 0=-\phi\left(\left\{1,2\right\},2\right)-\ln\left[\frac{v_H\left(2\right)/\sigma}{x\left(\left\{1,2\right\},2\right)-\left(1-e^{-\phi\left(\left\{1,2\right\},1\right)}\right)x\left(\left\{1,2\right\},1\right)}\right] \\
\phi\left(\left\{2,1\right\},2\right) &: \quad 0=-\phi\left(\left\{2,1\right\},2\right)-\ln\left[\frac{v_H\left(2\right)/\sigma}{x\left(\left\{2,1\right\},2\right)-\left(1-e^{-\phi\left(\left\{2,1\right\},1\right)}\right)x\left(\left\{2,1\right\},1\right)}\right] \\
\phi\left(\left\{2,2\right\},2\right) &: \quad 0=-\phi\left(\left\{2,2\right\},2\right)-\ln\left[\frac{v_H\left(2\right)/\sigma}{x\left(\left\{2,2\right\},2\right)-\left(1-e^{-\phi\left(\left\{2,2\right\},1\right)}\right)x\left(\left\{2,2\right\},1\right)}\right]
\end{aligned}$$

The last set of conditions is

$$\begin{aligned}
v_H\left(1\right) &: \quad 0=\phi\left(\left\{1,1\right\},1\right)P_F(\{1,1\})+\phi\left(\left\{1,2\right\},1\right)P_F(\{1,2\}) \\
&\quad +\phi\left(\left\{2,1\right\},1\right)P_F(\{2,1\})+\phi\left(\left\{2,2\right\},1\right)P_F(\{2,2\})-\Gamma\left[v_H\left(1\right)\right]P_H\left(1\right) \\
v_H\left(2\right) &: \quad 0=\phi\left(\left\{1,1\right\},2\right)P_F(\{1,1\})+\phi\left(\left\{1,2\right\},2\right)P_F(\{1,2\}) \\
&\quad +\phi\left(\left\{2,1\right\},2\right)P_F(\{2,1\})+\phi\left(\left\{2,2\right\},2\right)P_F(\{2,2\})-\Gamma\left[v_H\left(2\right)\right]P_H\left(2\right).
\end{aligned}$$

Let this system of equations be denoted $F(X) = 0$ where

$$X = \begin{pmatrix} \phi(\{1,1\},1) \\ \phi(\{1,2\},1) \\ \phi(\{2,1\},1) \\ \phi(\{2,2\},1) \\ \phi(\{1,1\},2) \\ \phi(\{1,2\},2) \\ \phi(\{2,1\},2) \\ \phi(\{2,2\},2) \\ v_H(1) \\ v_H(2) \end{pmatrix}$$

and where the Jacobian, $J(X)$ is

$$J(X) = \begin{bmatrix}
-1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -\frac{1}{v_H(1)} & 0 \\
0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{1}{v_H(1)} & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & -\frac{1}{v_H(1)} & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & -\frac{1}{v_H(1)} & 0 \\
d(1,1) & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{1}{v_H(2)} \\
0 & d(1,2) & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -\frac{1}{v_H(2)} \\
0 & 0 & d(2,1) & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{1}{v_H(2)} \\
0 & 0 & 0 & d(2,2) & 0 & 0 & 0 & -1 & 0 & -\frac{1}{v_H(2)} \\
P_F(\{1,1\}) & P_F(\{1,2\}) & P_F(\{2,1\}) & P_F(\{2,2\}) & 0 & 0 & 0 & 0 & g(1) & 0 \\
0 & 0 & 0 & 0 & P_F(\{1,1\}) & P_F(\{1,2\}) & P_F(\{2,1\}) & P_F(\{2,2\}) & 0 & g(2)
\end{bmatrix}$$

and where

$$d(z,a) = \frac{-e^{-\phi(\{z,a\},1)} x(\{z,a\},1)}{x(\{z,a\},2) - \left(1 - e^{-\phi(\{z,a\},1)}\right) x(\{z,a\},1)}$$

and

$$g(a) = -\Gamma'[v_H(a)] P_H(a)$$